New Results on Interference Channels with Three User Pairs

Bernd Bandemer

University of California, San Diego
Information Theory and Applications Center

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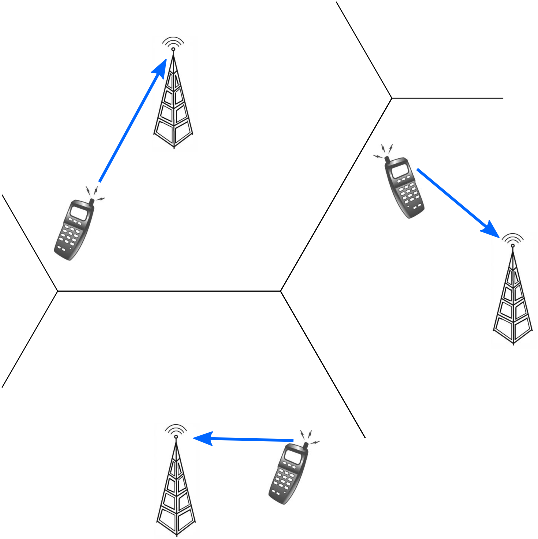
Interference channel



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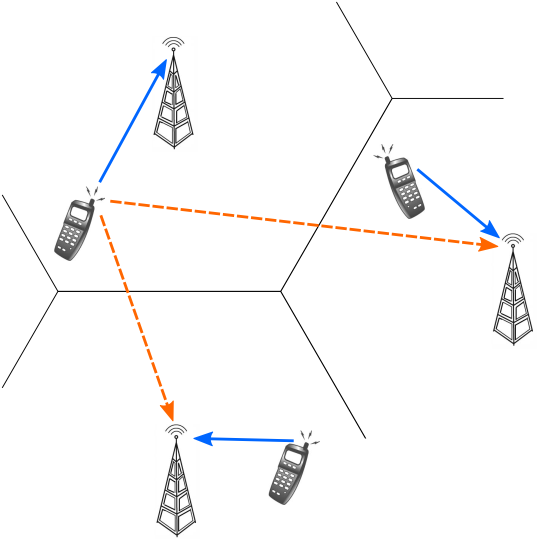

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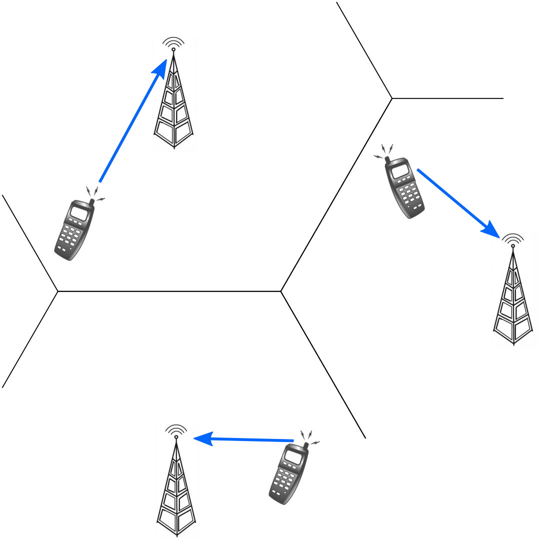

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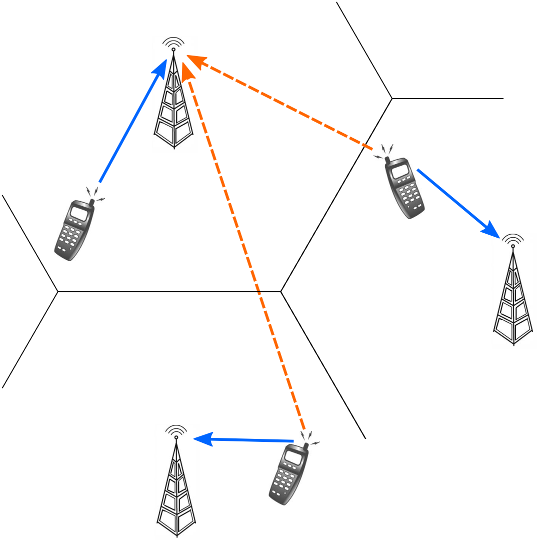

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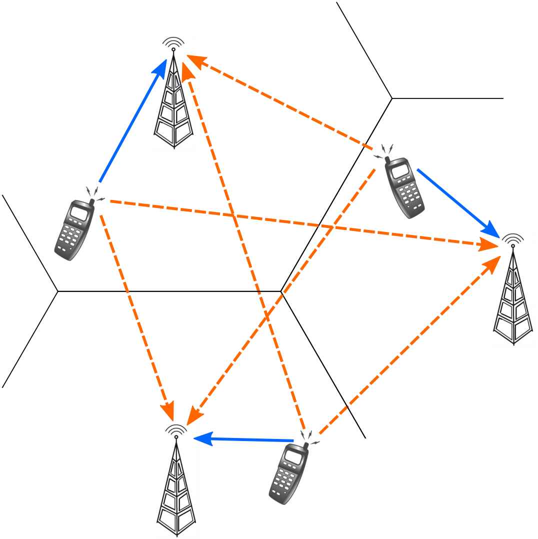

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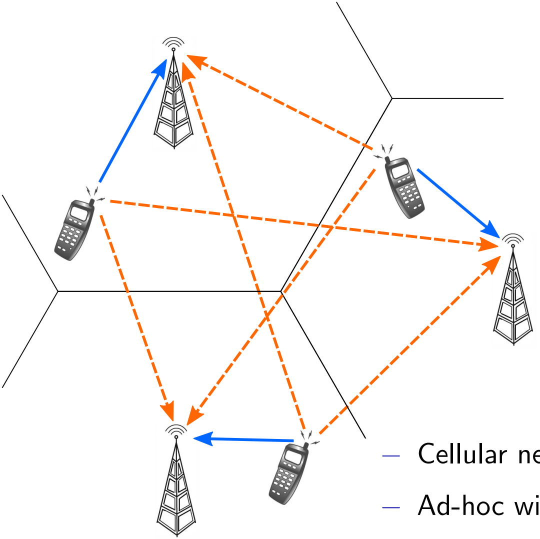
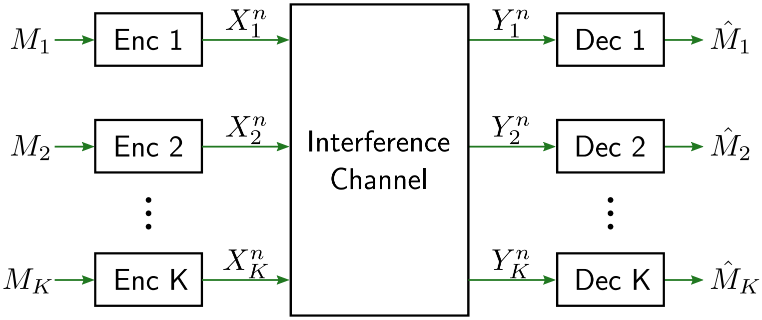


- Cellular networks
- Ad-hoc wireless networks
- Wireline: DSL

Interference channel



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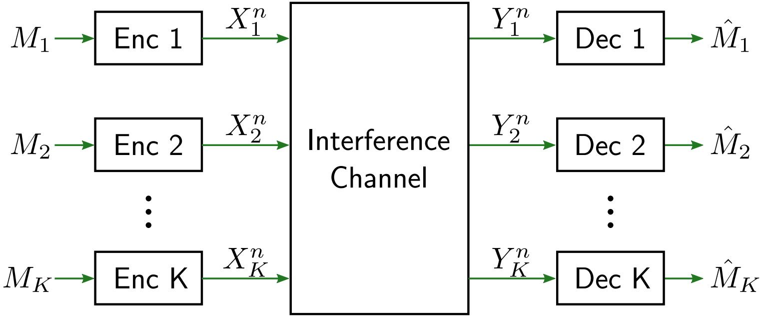

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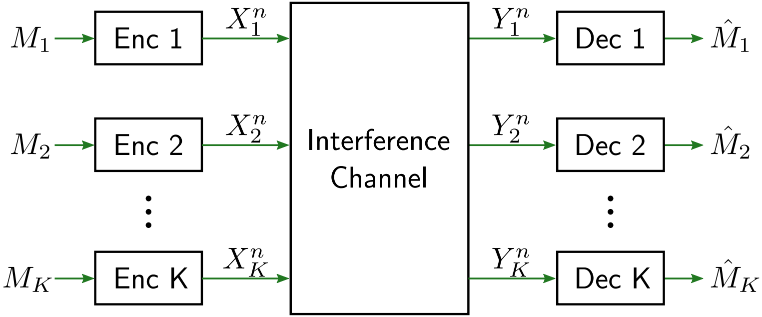

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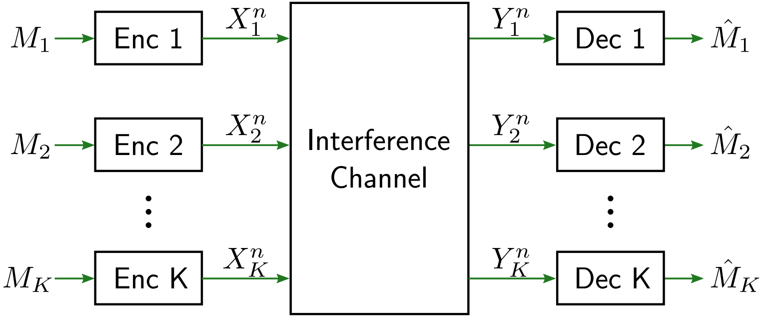

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- Best known scheme for two user pairs: *(Han–Kobayashi 81)*
- How to extend the Han–Kobayashi scheme to more user pairs?

Deterministic interference channels

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  - Channel noise
  - Interference

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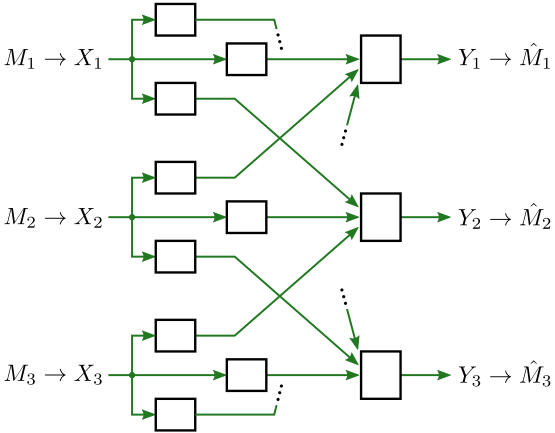
- **Deterministic interference channels**
  - No noise in the channel
  - Focus on signal interaction
  - Motivated by
    - Growing user density in current wireless networks
    - High-SNR regime of Gaussian channels *(Bresler–Tse 08)*
    - Study adverse effects separately

Deterministic interference channel (3-DIC)



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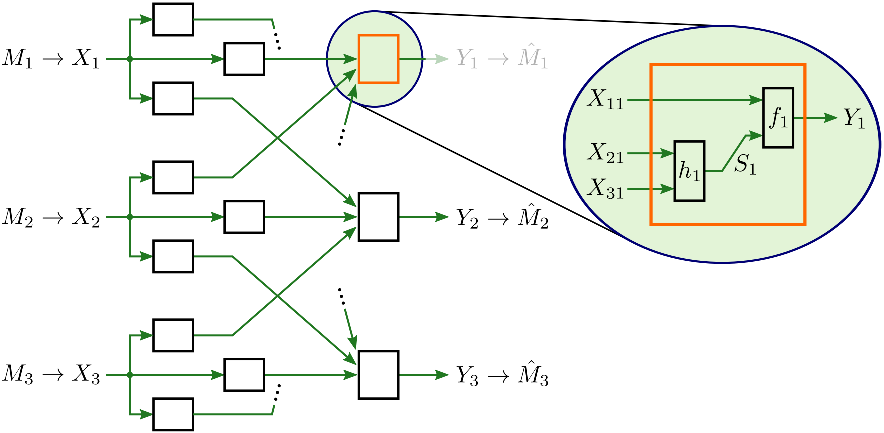

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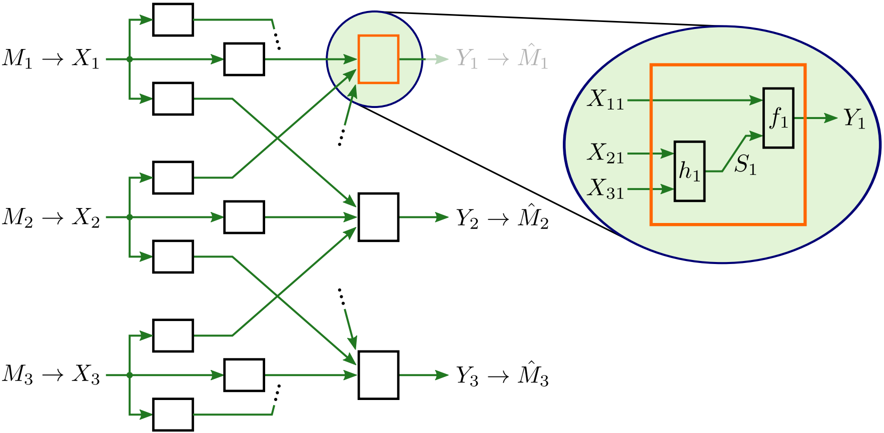

Injectivity: h_k and f_k are one-to-one in each argument

For f_1: $H(X_{11}) = H(Y_1 \mid S_1)$ and $H(S_1) = H(Y_1 \mid X_{11})$

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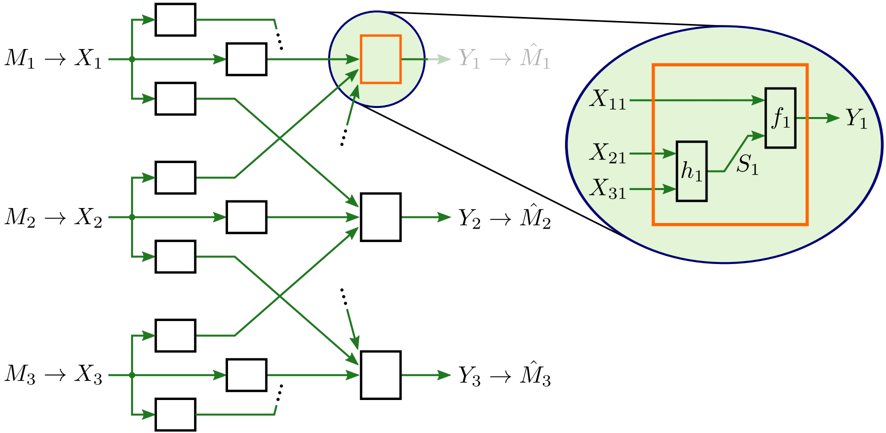

Define $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ code, probability of error, achievability of (R_1, R_2, R_3), and the capacity region in the usual way

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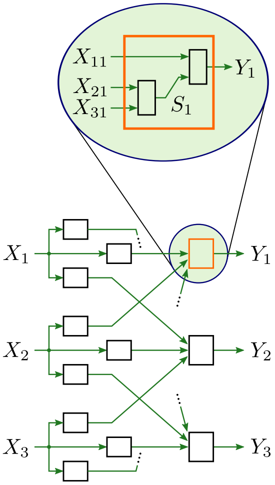

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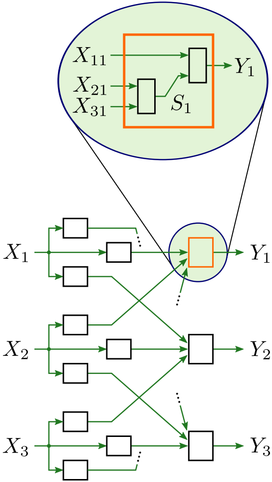

- Why this deterministic model?
  - 2-pair deterministic interference channel *(El Gamal–Costa 82)*
  - Fully invertible 3-DIC *(Gou–Jafar 09)*

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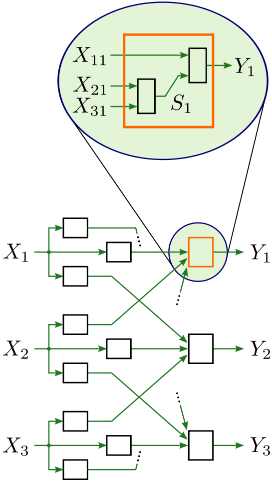

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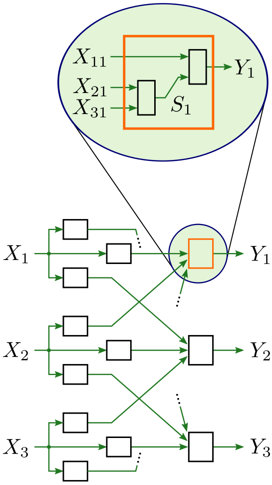

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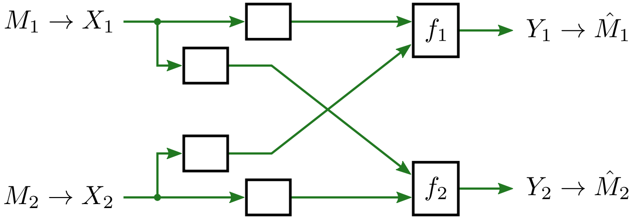
- Capacity region is not known in general
- We find a **new achievable rate region**
  - Includes previously known bounds
  - Naturally extends Han–Kobayashi

Two user pairs *(El Gamal–Costa 82)*



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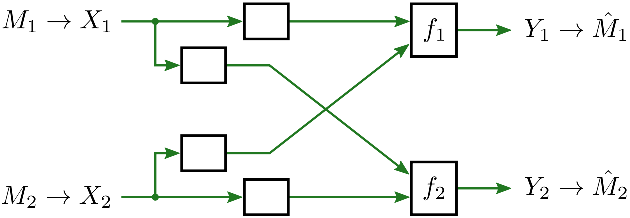

Coding strategies:

- Treat interference as noise

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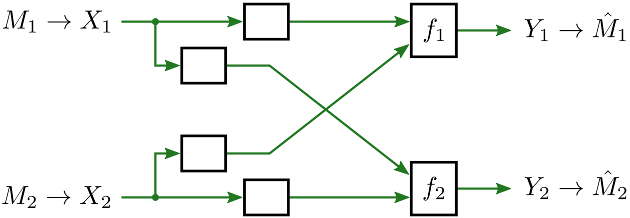

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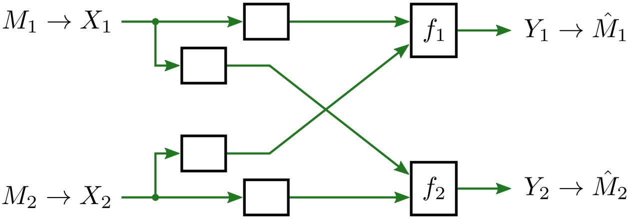

Coding strategies:

- Treat interference as noise
- Decode both messages
- Hybrid: partly decode, partly treat as noise *(Han–Kobayashi 81)*
  - Rate splitting and superposition coding
  - Achieves entire capacity region

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  (Maddah-Ali et al. 08, Cadambe–Jafar 08)
  - Constrain combined interference to a subspace
  - Disregard that subspace
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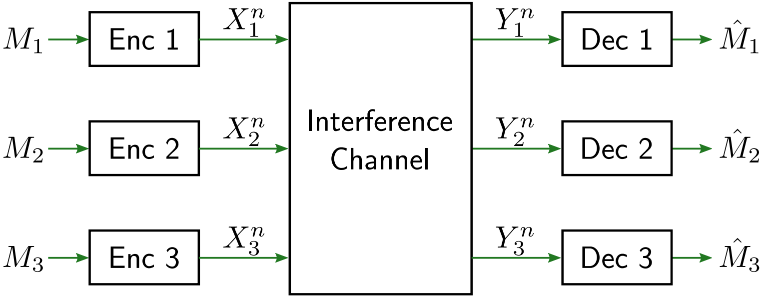
  - **Treat interference as noise**
- We are interested in **more general coding schemes**

Two aspects of interference channels



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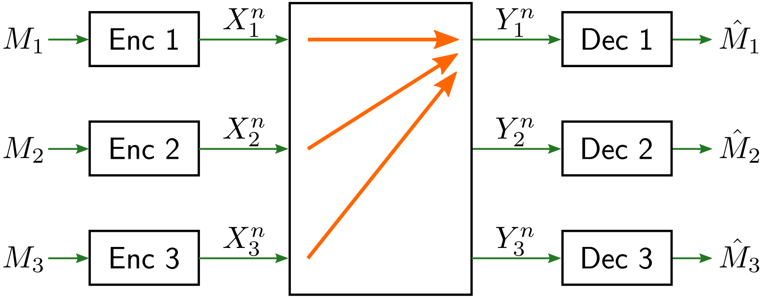

Multiple Access Channel

But: receiver is interested in only one message

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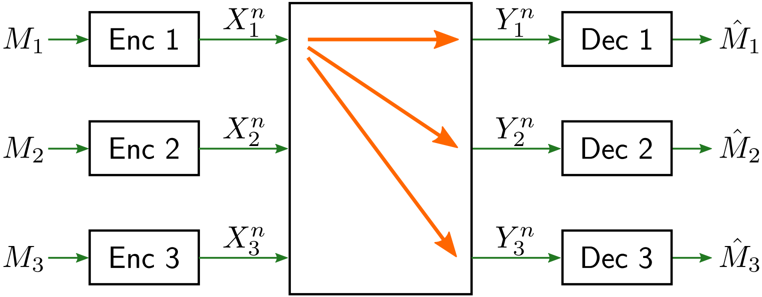


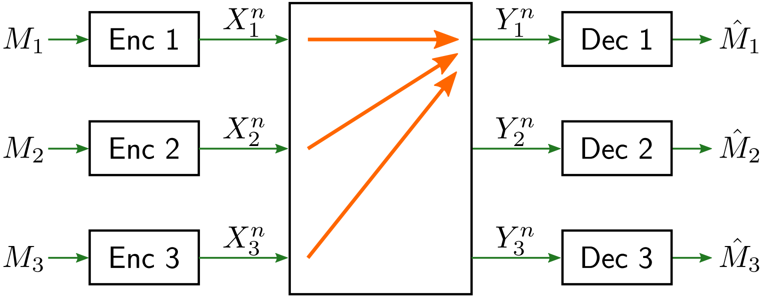
Broadcast channel

But: transmitter has a message for only one receiver

Talk outline

- Receiver-centric (MAC) aspect
  - Interference decoding
- Transmitter-centric (BC) aspect
  - Communication with disturbance constraints
- Combine the two aspects
  - Capacity inner bound for 3-DIC
- Extension to noisy channels

Receiver-centric aspect



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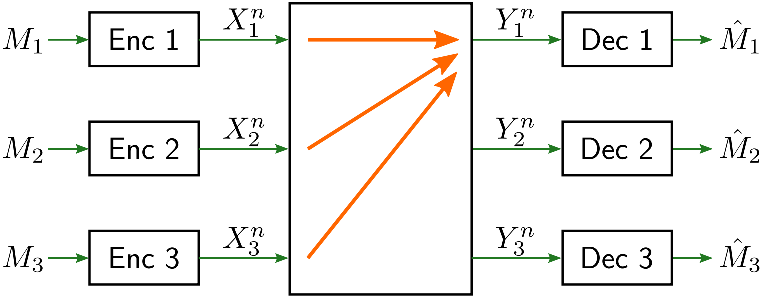

- Use simple point-to-point codes at the transmitters
- **Interference decoding**
  Receivers decode own message jointly with combined interference
  Simultaneous non-unique decoding

Interference-decoding inner bound *(B./El Gamal, ISIT 2010)*

Theorem

$$\mathscr{R}_{\mathrm{ID}} = \overline{\bigcup_{p} \mathscr{R}_1(p) \cap \mathscr{R}_2(p) \cap \mathscr{R}_3(p)},$$

where $(Q, X_1, X_2, X_3) \sim p = p(q)p(x_1|q)p(x_2|q)p(x_3|q)$, *is an inner bound to the capacity region of the 3-DIC.*

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- $\mathscr{R}_1(p)$ is the set of (R_1, R_2, R_3) that satisfy

$$\begin{aligned} R_1 &< H(X_{11} \,|\, Q), \\ R_1 + \min\{R_2, H(X_{21} \,|\, Q)\} &< H(Y_1 \,|\, X_{31}, Q), \\ R_1 + \min\{R_3, H(X_{31} \,|\, Q)\} &< H(Y_1 \,|\, X_{21}, Q), \\ R_1 + \min\{R_2 + R_3, H(S_1 \,|\, Q), R_2 + H(X_{31} \,|\, Q),& \\ H(X_{21} \,|\, Q) + R_3\} &< H(Y_1 \,|\, Q) \end{aligned}$$

- Likewise, $\mathscr{R}_2$ and $\mathscr{R}_3$

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Remarks:

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  - Interference alignment

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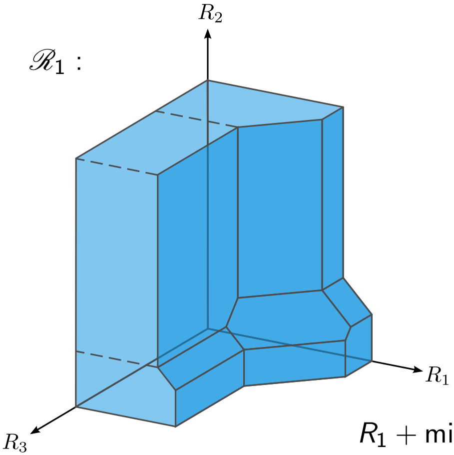
Remarks:

- Generally larger than interference-as-noise inner bound
  - Interference alignment
- Achieves capacity in some cases
  - Sum capacity of 3-user Q-ary extension channel *(B. et al. ISIT 2009)*
  - Strong interference, invertible h_k

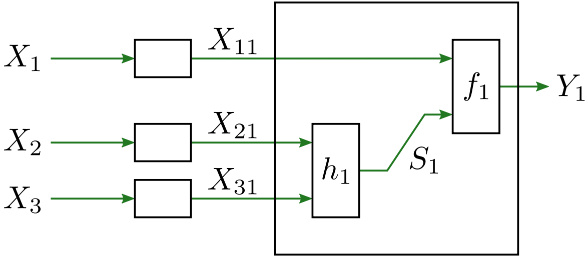
General shape of $\mathscr{R}_1$



- $\mathscr{R}_1$ is unbounded in R_2 and R_3
  - $R_1 > 0$ regardless of R_2, R_3
- As R_2, R_3 decrease, S_1 becomes more structured
  - Helps increase R_1

$$\begin{aligned} R_1 &< H(X_{11} \mid Q), \\ R_1 + \min\{R_2, H(X_{21} \mid Q)\} &< H(Y_1 \mid X_{31}, Q), \\ R_1 + \min\{R_3, H(X_{31} \mid Q)\} &< H(Y_1 \mid X_{21}, Q), \\ R_1 + \min\{R_2 + R_3, H(S_1 \mid Q), R_2 + H(X_{31} \mid Q),& \\ H(X_{21} \mid Q) + R_3\} &< H(Y_1 \mid Q) \end{aligned}$$

Saturation effect



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$$

Proof of achievability

- **Codebook generation**
  Fix $p(x_1)p(x_2)p(x_3)$
  Generate $x_1^n(m_1) \sim \prod_{i=1}^n p_{X_1}(x_{1i})$, for $m_1 \in [1:2^{nR_1}]$
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  To send m_1, transmit $x_1^n(m_1)$. Likewise for other users
- **Decoding:** Simultaneous non-unique decoding
  Observe y_1^n. Find unique $\hat{m}_1$ such that

$$(x_1^n(\hat{m}_1), s_1^n(m_2, m_3), x_{21}^n(m_2), x_{31}^n(m_3), y_1^n) \in \mathcal{T}_\varepsilon^{(n)}$$

  for some m_2, m_3
  Likewise for other users

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- Interference decoding inner bound not optimal in general

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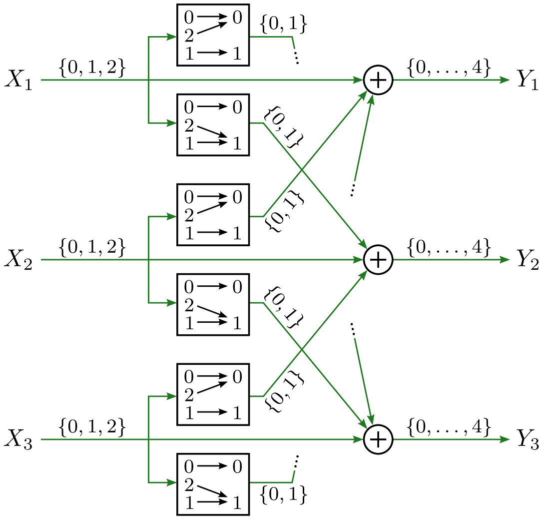
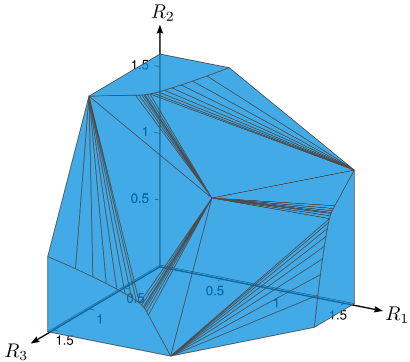
    For fixed codebook structure, the decoder structure is optimal
- Receiver-centric aspect is solved
  - Simultaneous non-unique decoding makes optimal use of codebook knowledge
- In fact, this holds for general K-pair discrete memoryless ICs
  (B./Kim/El Gamal, manuscript in preparation)

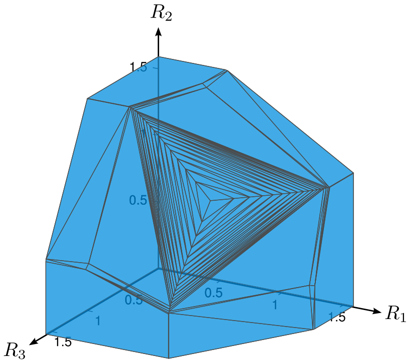
Example



Example: Achievable rate regions

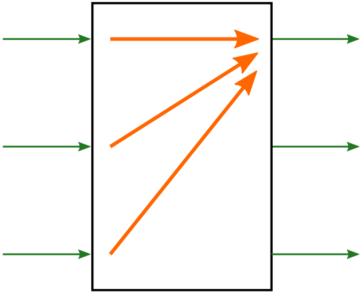


Interference as noise



Interference decoding

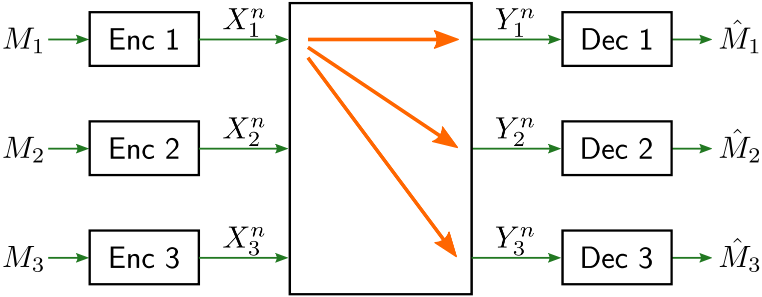
Summary – Receiver-centric aspect



Interference decoding

- Improves upon treating interference as noise
- Given the codebook structure, simultaneous non-unique decoding is optimal
- Fully exploits structure of combined interference signal

Transmitter-centric aspect



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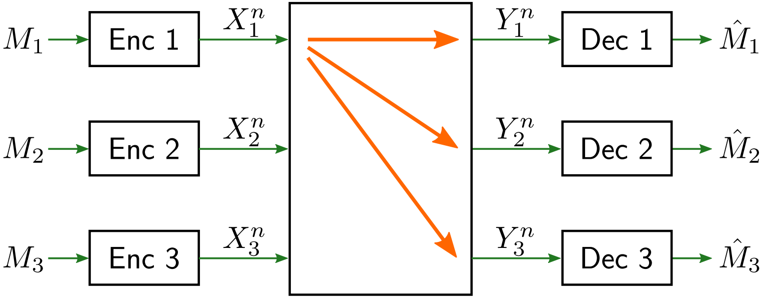
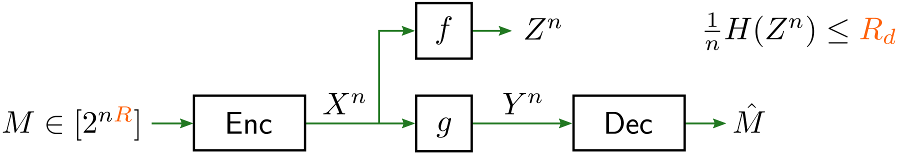


- We define **disturbance-constrained communication**

Disturbance-constrained communication *(B./El Gamal, ISIT 2011)*



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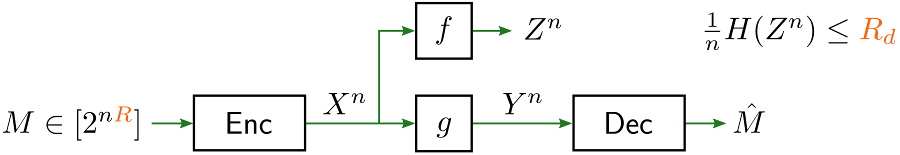
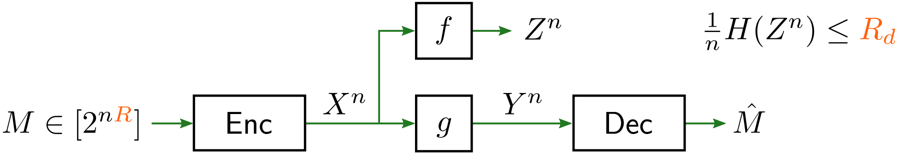


Rate–disturbance trade-off

- Pair (R, R_d) is achievable:
  Sequence of $(2^{nR}, n)$ codes exists with

$$\lim_{n\to\infty} \mathrm{P}(\hat{M} \neq M) = 0 \qquad \limsup_{n\to\infty} \frac{1}{n} H(Z^n) \le R_d$$

Disturbance-constrained communication *(B./El Gamal, ISIT 2011)*



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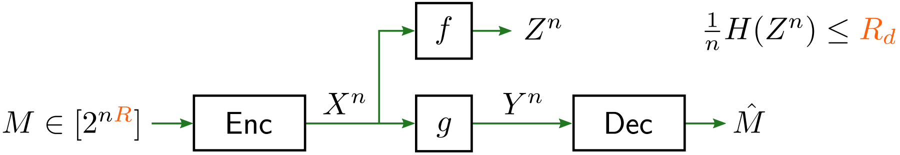
- Pair (R, R_d) is achievable:
  Sequence of $(2^{nR}, n)$ codes exists with

$$\lim_{n\to\infty} \mathrm{P}(\hat{M} \neq M) = 0 \qquad \limsup_{n\to\infty} \frac{1}{n} H(Z^n) \le R_d$$

- **Rate–disturbance region $\mathscr{R}$**
  Closure of all achievable rate–disturbance pairs

Result: Rate–disturbance region



Theorem

The rate–disturbance region $\mathscr{R}$ is the set of pairs $(R, R_d) \in \mathbb{R}_+^2$ satisfying

$$R \le H(Y)$$
$$R - R_d \le H(Y \mid Z)$$

for some distribution $p(x)$

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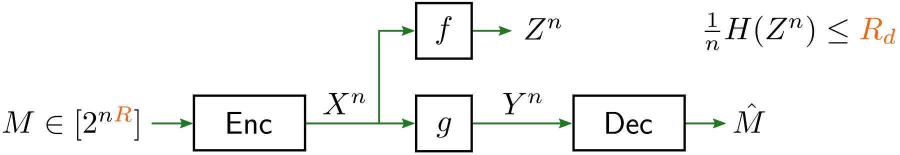

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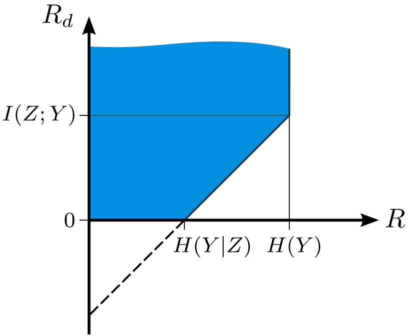

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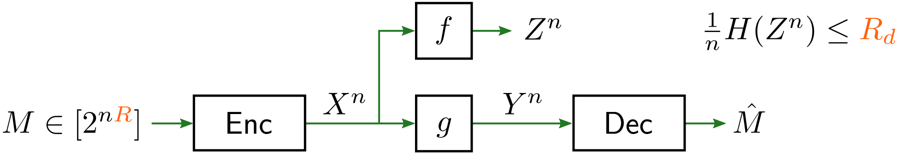

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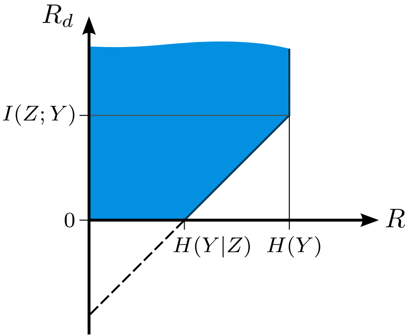
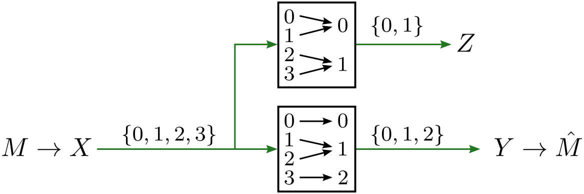


(This theorem extends to noisy channels.)

Example



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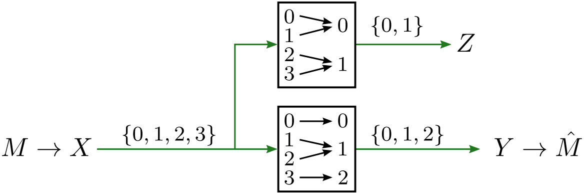
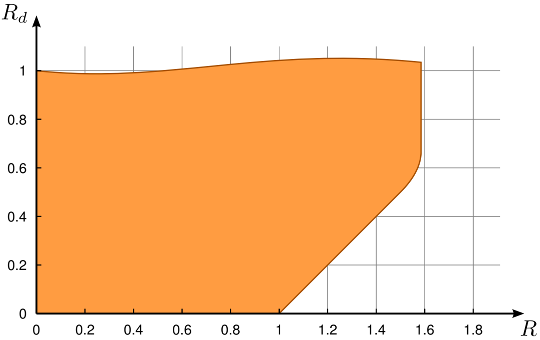

$M \to X$
$\{0,1,2,3\}$
0
1
2
3
0
1
$\{0,1\}$
Z
0
1
2
3
0
1
2
$\{0,1,2\}$
$Y \to \hat{M}$


R_d
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
R

Achievability proof sketch

- **Rate splitting**: $R = R_0 + R_1$
- **Superposition coding:**

  Fix $p(x)$. This induces $p(z)p(x|z)$
  Generate 2^{nR_0} cloud centers $\sim p(z)$
  Generate 2^{nR_1} satellite codewords $\sim p(x|z)$

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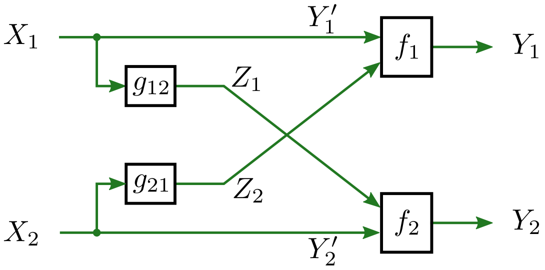
  Generate 2^{nR_0} cloud centers $\sim p(z)$
  Generate 2^{nR_1} satellite codewords $\sim p(x|z)$
- **Analysis:**
  Side receiver distinguishes cloud centers, but not satellite codewords

Connection to interference channel

Injective deterministic **interference channel** *(El Gamal–Costa 1982)*



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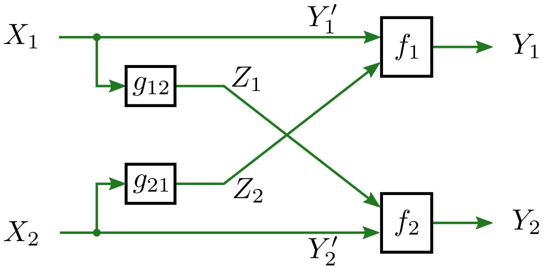

- Optimal scheme: **Han–Kobayashi** with $U_1 = Z_1$, $U_2 = Z_2$

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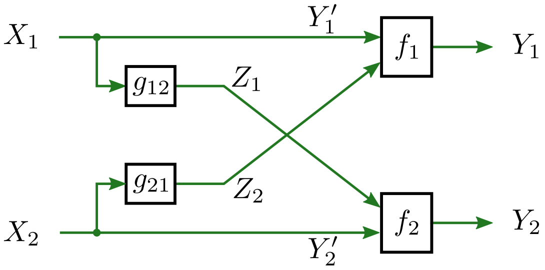

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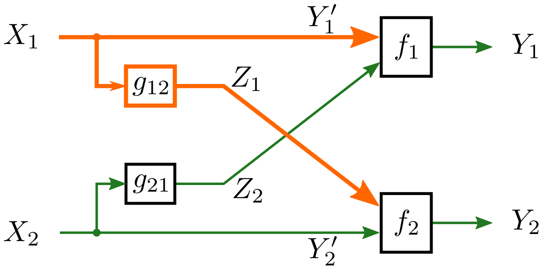


- Optimal scheme: **Han–Kobayashi** with $U_1 = Z_1,\ U_2 = Z_2$
  - is rate-splitting and superposition coding
  - coincides with **disturbance-minimizing scheme**

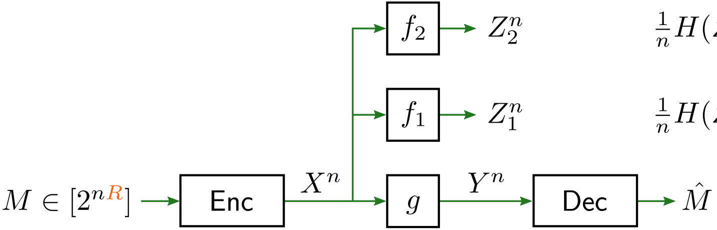
Extension to two disturbance constraints

One disturbance constraint	$\leftrightarrow$	**2-pair interference channel**
Optimal scheme		Han–Kobayashi scheme

Extension to two disturbance constraints

One disturbance constraint Optimal scheme	$\leftrightarrow$	**2-pair interference channel** Han–Kobayashi scheme
Two disturbance constraints	$\rightarrow$	**3-pair interference channel**

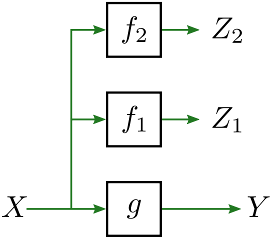
Two disturbance constraints



$$\frac{1}{n}H(Z_2^n) \leq R_{d,2}$$

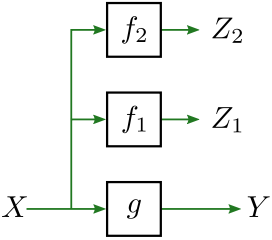
$$\frac{1}{n}H(Z_1^n) \leq R_{d,1}$$

Coding scheme



Coding scheme

Split rate $R = R_0 + R_1 + R_2 + R_3$
Let $\tilde{R}_1 \geq R_1$ and $\tilde{R}_2 \geq R_2$



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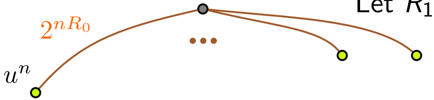

- u^n i.i.d.

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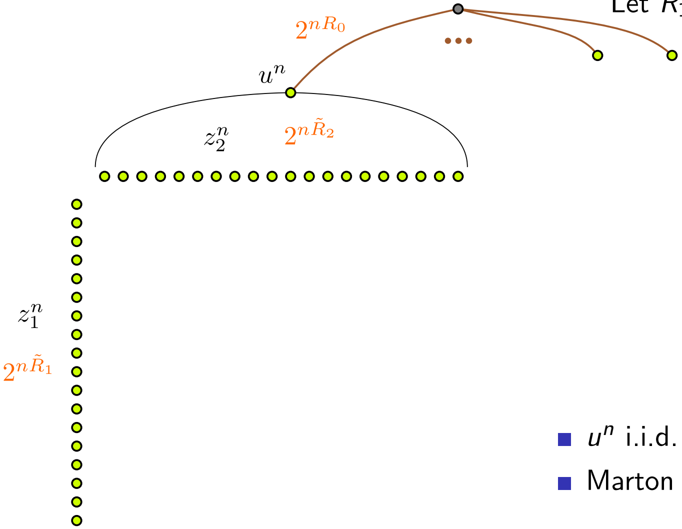

- u^n i.i.d.
- Marton (z_1^n, z_2^n) given u^n

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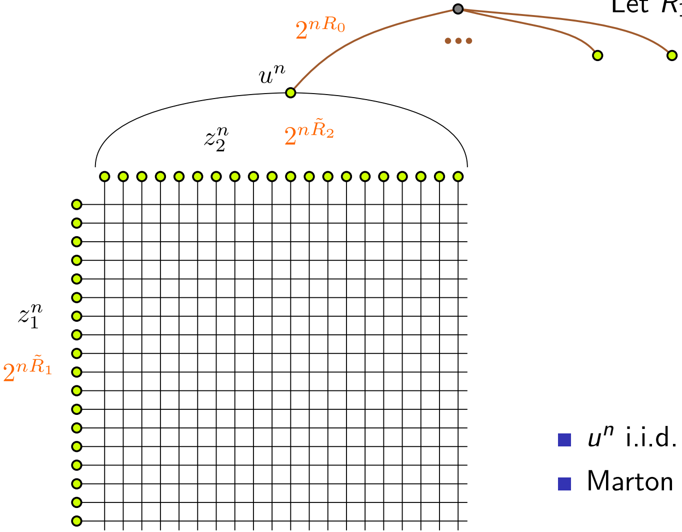

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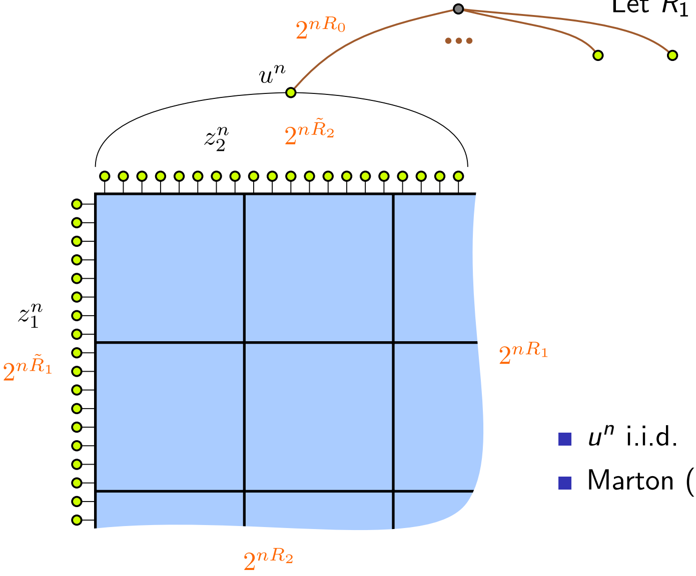

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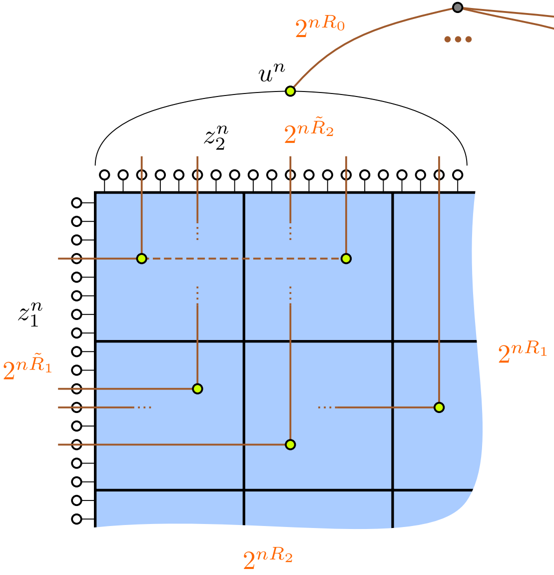

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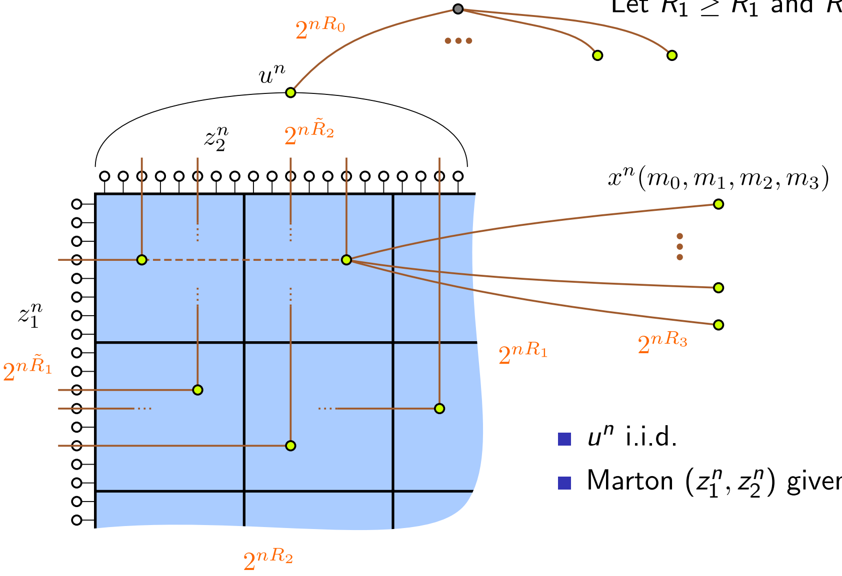

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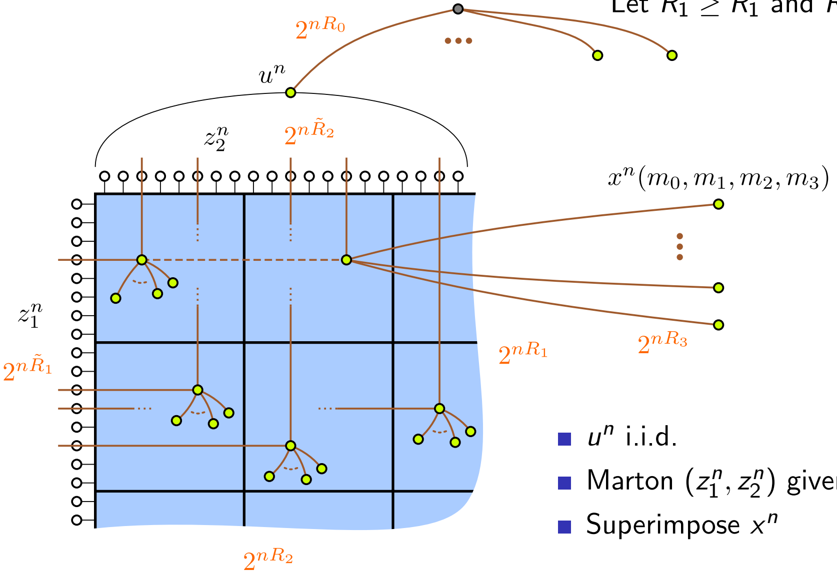

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- Superimpose x^n

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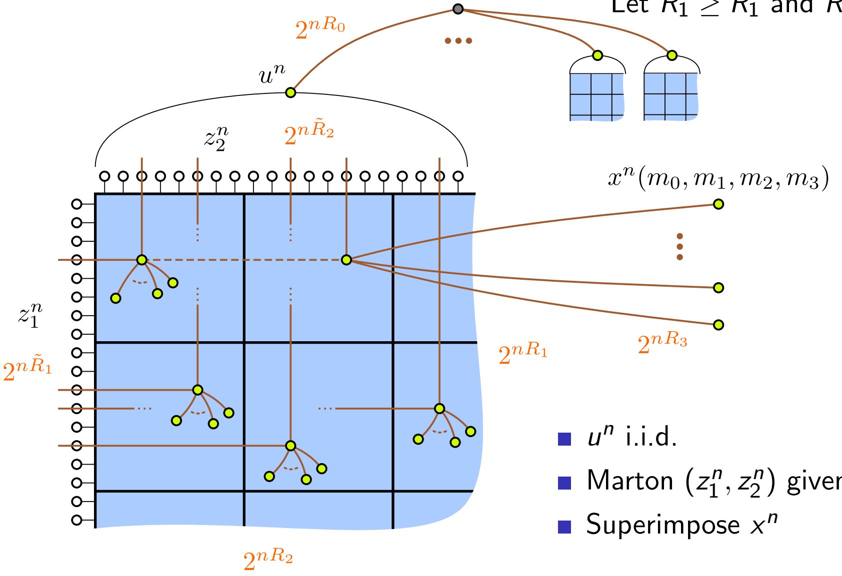

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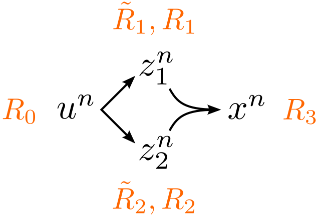
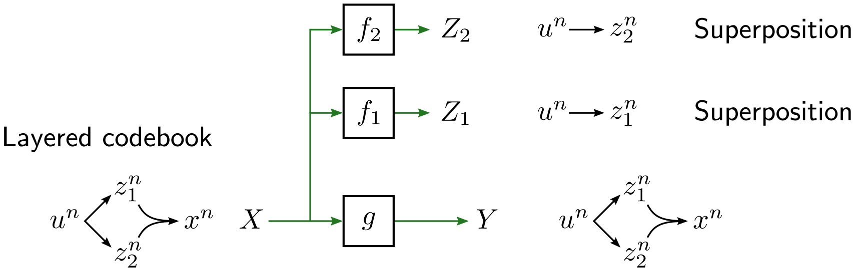


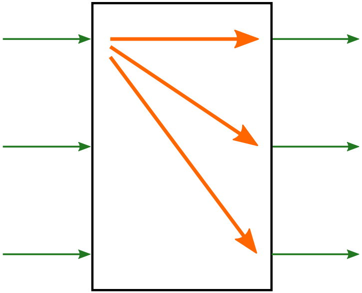
Summary – Two disturbance constraints

- Use **Marton coding and superposition coding**

Transmitted: **Observed structure:**



Summary – Transmitter-centric aspect

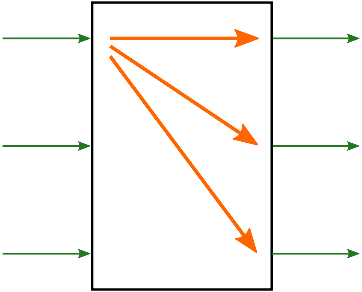


Disturbance-constrained communication

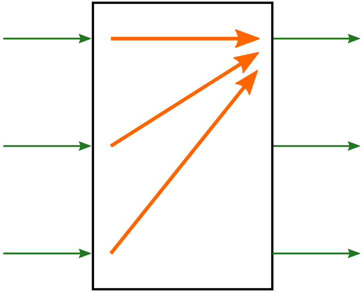
- Link to interference channels (single constraint case recovers Han-Kobayashi)
- Two constraints: Layered coding scheme

Putting the pieces together — 3-DIC

BC aspect



MAC aspect



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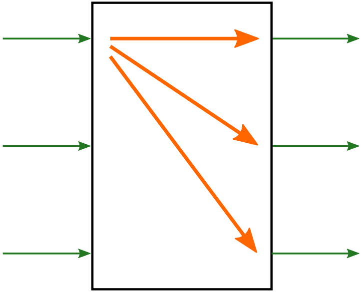
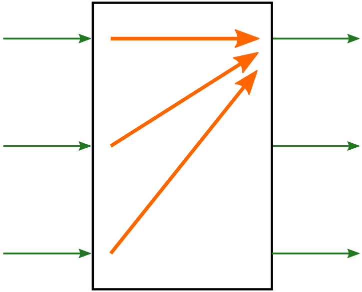


Marton coding and superposition coding

$+$

MAC aspect



Interference decoding

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- Codebooks as in **disturbance-constrained communication**
  Fixed distributions $p(u_k, x_k)$ for all users k

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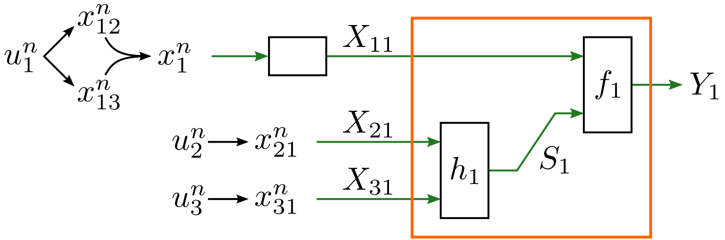

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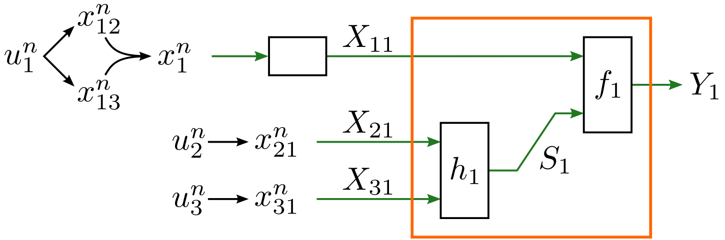

- Receivers use **interference decoding**
  Use full knowledge of interfering codebooks
  Non-unique simultaneous decoding

3-DIC capacity region inner bound

Theorem

$$\mathscr{R} = \mathrm{FM}\left\{\overline{\bigcup_{p} \mathscr{R}_1(p) \cap \mathscr{R}_2(p) \cap \mathscr{R}_3(p)}\right\},$$

where p is of the form $p = p(q)p(u_1, x_1|q)p(u_2, x_2|q)p(u_3, x_3|q)$, is an inner bound to the capacity region of the 3-DIC.

Where

- FM is a specialized Fourier–Motzkin elimination
- $\mathscr{R}_1$, $\mathscr{R}_2$, and $\mathscr{R}_3$ are rate regions in $\mathbb{R}^{18}$

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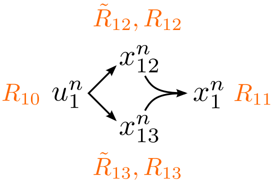

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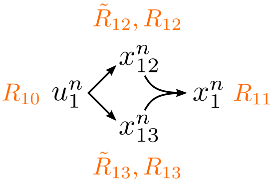
  Details are quite involved ...

  But easy to evaluate by computer



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Remarks:

- Strictly contains interference-decoding inner bound
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3-DIC capacity region inner bound

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Remarks:

- Strictly contains interference-decoding inner bound
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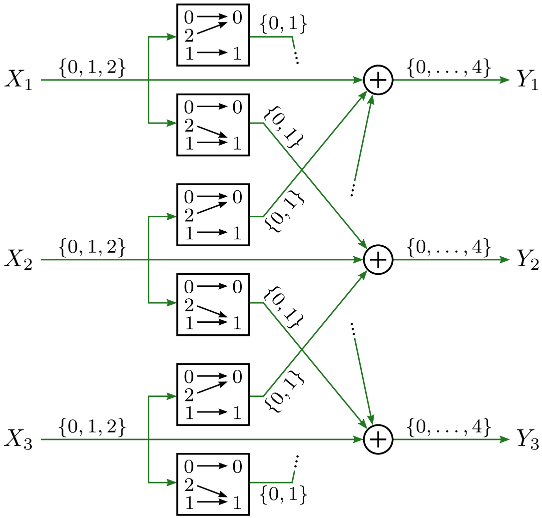
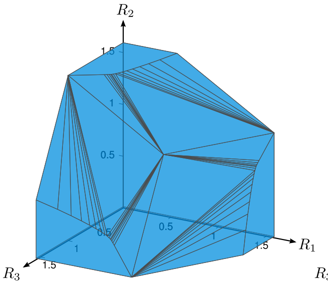
- Achieves 2-pair marginal capacities
  - Subsumes Han-Kobayashi scheme for every 2-pair subchannel

Example

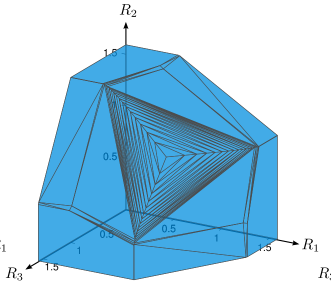


X_1
$\{0,1,2\}$
Y_1
$\{0,\ldots,4\}$
X_2
$\{0,1,2\}$
Y_2
$\{0,\ldots,4\}$
X_3
$\{0,1,2\}$
Y_3
$\{0,\ldots,4\}$
$\{0,1\}$
$0 \to 0$
$2 \to 0$
$1 \to 1$
$0 \to 0$
$2 \to 1$
$1 \to 1$

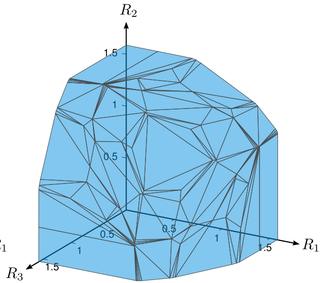
Example: Rate regions



Interference as noise

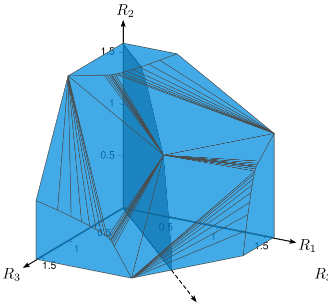


Interference decoding



Layered scheme &
Interference Decoding

Example: Rate regions



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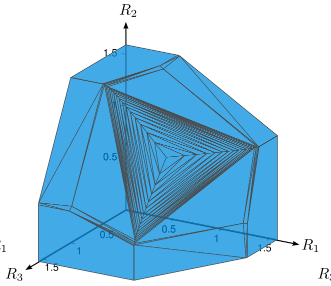
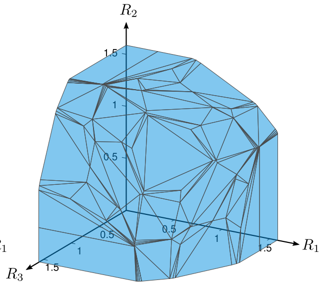
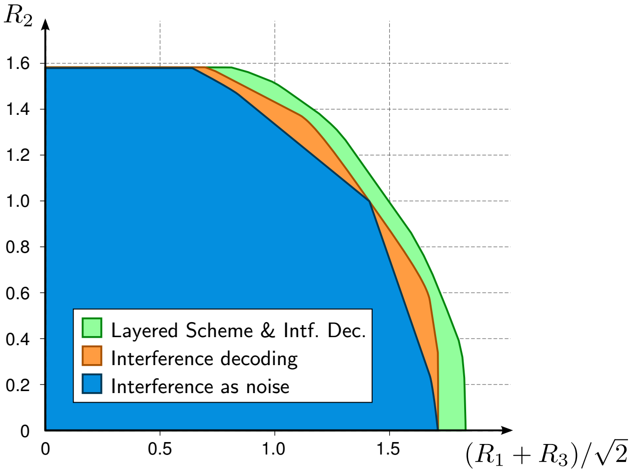


Interference decoding

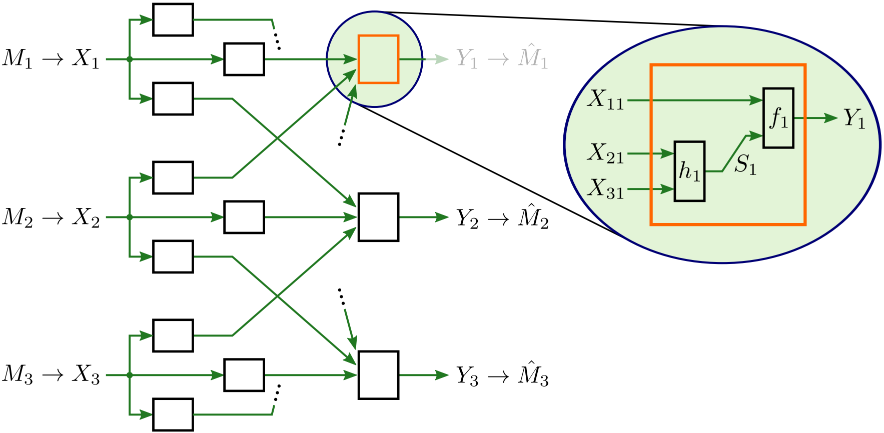


Layered scheme & Interference Decoding

Example: Rate regions

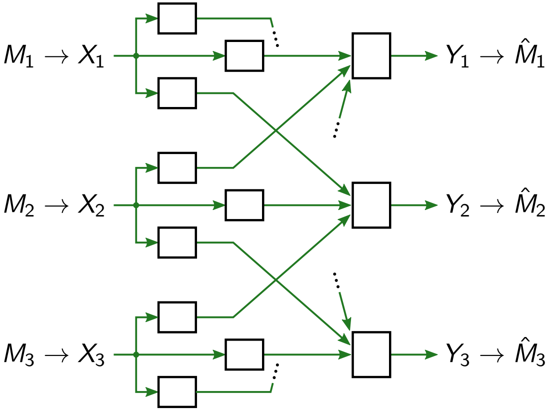


Summary – Achievable rate region for 3-DIC



- New inner bound to the capacity region
  - Non-symbolic Fourier–Motzkin elimination
  - Numerous modes of saturation

Extension to channels with noise *(to be presented at ISIT 2012)*



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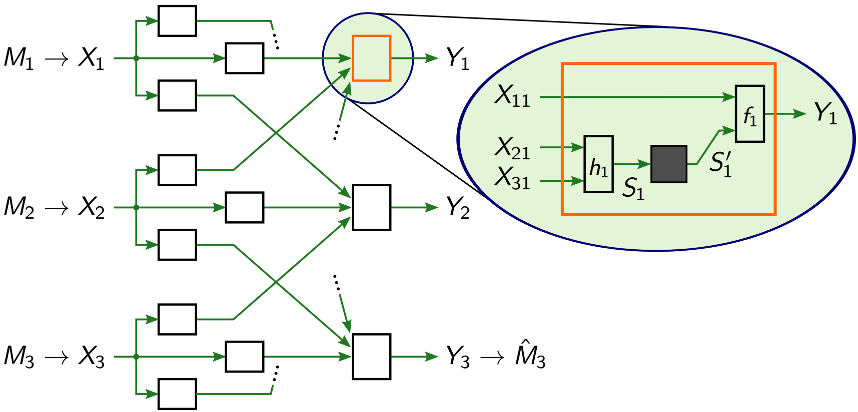
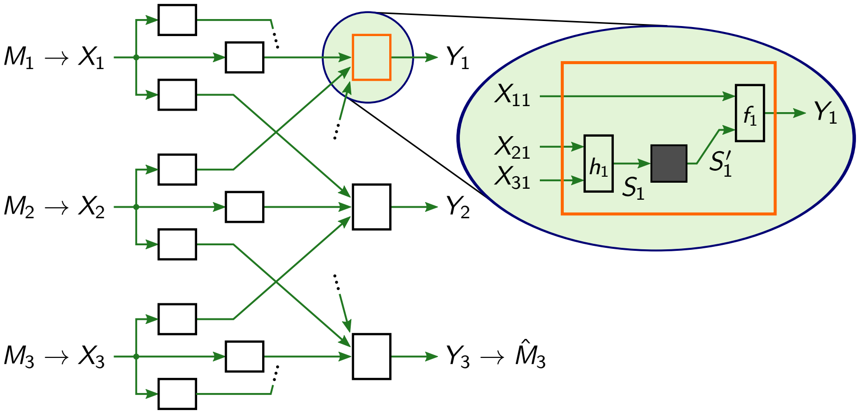


- Combined interference S_k passes through a noisy channel $S_k \to S_k'$
  - Arbitrary DMC $p(s_k'|s_k)$
- h_k and f_k remain injective in each argument

Extension to channels with noise *(to be presented at ISIT 2012)*



This channel:

- Has structure similar to 3-DIC
- Contains the Gaussian IC as a special case

Inner bound for channels with noise

Theorem

$$\mathscr{R} = \mathrm{FM}\left\{\overline{\bigcup_p \mathscr{R}_1(p) \cap \mathscr{R}_2(p) \cap \mathscr{R}_3(p)}\right\},$$

where p is of the form $p = p(q)p(u_1, x_1|q)p(u_2, x_2|q)p(u_3, x_3|q)$, is an inner bound to the capacity region of the 3-DIC.

As before,

- FM is a specialized Fourier–Motzkin elimination
- $\mathscr{R}_1$, $\mathscr{R}_2$, and $\mathscr{R}_3$ are rate regions in $\mathbb{R}^{18}$

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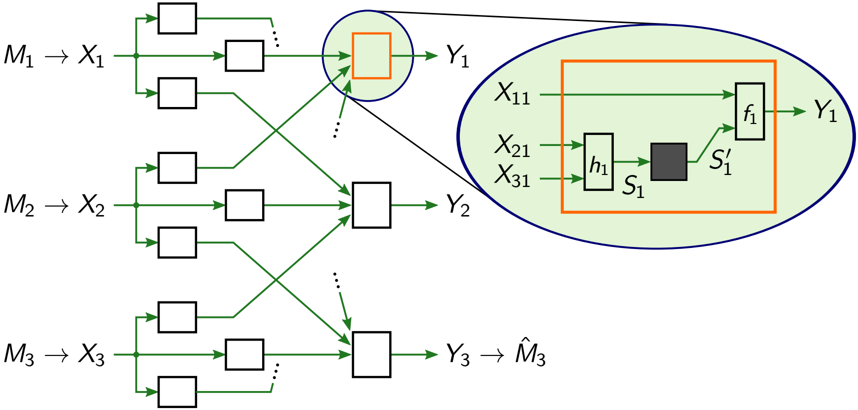
As before,

- FM is a specialized Fourier–Motzkin elimination
- $\mathscr{R}_1$, $\mathscr{R}_2$, and $\mathscr{R}_3$ are rate regions in $\mathbb{R}^{18}$

  Details are **still** quite involved ...

  But **still** easy to evaluate by computer

Summary – Channels with noise



- Inner bound techniques continue to work
- Achievable regimes subsumes Han–Kobayashi scheme

Conclusion

- Receiver-centric aspect
  - Interference decoding: Exploit structure of combined interference
  - Simultaneous non-unique decoding is optimal
  - Subsumes treating interference as noise

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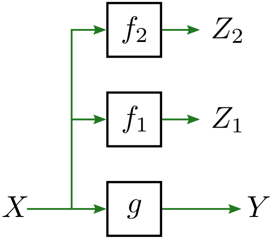
- Transmitter-centric aspect
  - Introduced communication with disturbance constraints
  - Found rate–disturbance region for single constraint
  - Connection to interference channels
  - Inner bound for deterministic case with two constraints

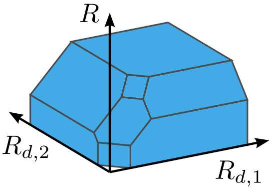
Conclusion

- Receiver-centric aspect
  - Interference decoding: Exploit structure of combined interference
  - Simultaneous non-unique decoding is optimal
  - Subsumes treating interference as noise
- Transmitter-centric aspect
  - Introduced communication with disturbance constraints
  - Found rate–disturbance region for single constraint
  - Connection to interference channels
  - Inner bound for deterministic case with two constraints
- Combination of the two aspects
  - New capacity inner bound for 3-DIC: improves previous bounds
  - Transmission scheme extends to noisy channels

Thanks!

Inner bound on the rate–disturbance region





Theorem

The set of $(R, R_{d,1}, R_{d,2}) \in \mathbb{R}_+^3$ *satisfying*

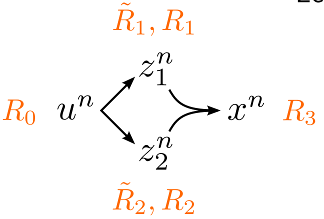
$$\begin{aligned}
R &\le H(Y) \\
R_{d,1} + R_{d,2} &\ge I(Z_1; Z_2 \,|\, U) \\
R - R_{d,1} &\le H(Y \,|\, Z_1, U) \\
R - R_{d,2} &\le H(Y \,|\, Z_2, U) \\
R - R_{d,1} - R_{d,2} &\le H(Y \,|\, Z_1, Z_2, U) \\
&\qquad - I(Z_1; Z_2 \,|\, U) \\
2R - R_{d,1} - R_{d,2} &\le H(Y \,|\, Z_1, Z_2, U) + H(Y \,|\, U) \\
&\qquad - I(Z_1; Z_2 \,|\, U)
\end{aligned}$$

for some joint distribution $p(u, x)$*, is an inner bound to the rate–disturbance region*

(This is optimal in some cases, e.g., if $Y = X$)

Coding scheme

Split rate $R = R_0 + R_1 + R_2 + R_3$
Let $\tilde{R}_1 \geq R_1$ and $\tilde{R}_2 \geq R_2$



Decoding conditions:

$$
\begin{aligned}
R_3 &< H(Y \mid Z_1, Z_2, U) \\
\tilde{R}_1 + R_3 &< H(Y \mid Z_2, U) + I(Z_1; Z_2 \mid U) \\
\tilde{R}_2 + R_3 &< H(Y \mid Z_1, U) + I(Z_1; Z_2 \mid U) \\
\tilde{R}_1 + \tilde{R}_2 + R_3 &< H(Y \mid U) + I(Z_1; Z_2 \mid U) \\
R_0 + \tilde{R}_1 + \tilde{R}_2 + R_3 &< H(Y) + I(Z_1; Z_2 \mid U)
\end{aligned}
$$

(plus some encoding conditions)

3-DIC inner bound — Conditions for $\mathscr{R}_1$ (first receiver)

Conditions: Encoder conditions, and

$$r_1 + \min\{r_{21} + r_{31}, H(S_1 \mid c_{21}, c_{31})\} < H(Y_1 \mid c_1, c_{21}, c_{31}) + t_1$$

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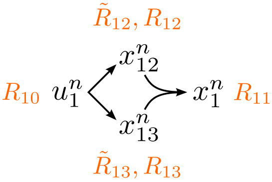
From disturbance-constrained communication

r_1	c_1	t_1
R_{11}	$U_1,\ X_{12},\ X_{13}$	0
$\tilde{R}_{12} + R_{11}$	$U_1,\ X_{13}$	$I(X_{12}; X_{13} \mid U_1)$
$\tilde{R}_{13} + R_{11}$	$U_1,\ X_{12}$	$I(X_{12}; X_{13} \mid U_1)$
$\tilde{R}_{12} + \tilde{R}_{13} + R_{11}$	U_1	$I(X_{12}; X_{13} \mid U_1)$
$R_{10} + \tilde{R}_{12} + \tilde{R}_{13} + R_{11}$	$\emptyset$	$I(X_{12}; X_{13} \mid U_1)$



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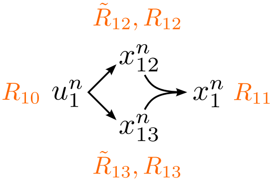

From interference decoding

r_{21}	c_{21}
0	X_{21}
$\min\{\tilde{R}_{21}, H(X_{21} \mid U_2)\}$	U_2
$\min\{R_{20} + \tilde{R}_{21},$ $R_{20} + H(X_{21} \mid U_2), H(X_{21})\}$	$\emptyset$

R_{20} $u_2^n \longrightarrow x_{21}^n$ $\tilde{R}_{21}$

3-DIC inner bound — Conditions for $\mathscr{R}_1$ (first receiver)

Conditions: Encoder conditions, and

$$r_1 + \min\{r_{21} + r_{31}, H(S_1 \mid c_{21}, c_{31})\} < H(Y_1 \mid c_1, c_{21}, c_{31}) + t_1$$

From disturbance-constrained communication

r_1	c_1	t_1
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$\tilde{R}_{12} + R_{11}$	$U_1,\ X_{13}$	$I(X_{12}; X_{13} \mid U_1)$
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$R_{10} + \tilde{R}_{12} + \tilde{R}_{13} + R_{11}$	$\emptyset$	$I(X_{12}; X_{13} \mid U_1)$

$$\begin{tikzcd}
 & \tilde{R}_{12}, R_{12} & & \\
 & x_{12}^n \arrow[dr] & & \\
R_{10}\ u_1^n \arrow[ur] \arrow[dr] & & x_1^n\ R_{11} & \\
 & x_{13}^n \arrow[ur] & & \\
 & \tilde{R}_{13}, R_{13} & &
\end{tikzcd}$$

From interference decoding

r_{21}	c_{21}
0	X_{21}
$\min\{\tilde{R}_{21}, H(X_{21} \mid U_2)\}$	U_2
$\min\{R_{20} + \tilde{R}_{21}, R_{20} + H(X_{21} \mid U_2), H(X_{21})\}$	$\emptyset$

r_{31}	c_{31}
0	X_{31}
$\min\{\tilde{R}_{31}, H(X_{31} \mid U_3)\}$	U_3
$\min\{R_{30} + \tilde{R}_{31}, R_{30} + H(X_{31} \mid U_3), H(X_{31})\}$	$\emptyset$

$$R_{20}\quad u_2^n \longrightarrow x_{21}^n\quad \tilde{R}_{21}$$

$$R_{30}\quad u_3^n \longrightarrow x_{31}^n\quad \tilde{R}_{31}$$

3-DIC inner bound — Conditions for $\mathscr{R}_1$ (first receiver)

Conditions: Encoder conditions, and

$$r_1 + \min\{r_{21} + r_{31}, H(S_1 \mid c_{21}, c_{31})\} < H(Y_1 \mid c_1, c_{21}, c_{31}) + t_1$$

From disturbance-constrained communication

r_1	c_1	t_1
R_{11}	$U_1,\ X_{12},\ X_{13}$	0
$\tilde{R}_{12} + R_{11}$	$U_1,\ X_{13}$	$I(X_{12}; X_{13} \mid U_1)$
$\tilde{R}_{13} + R_{11}$	$U_1,\ X_{12}$	$I(X_{12}; X_{13} \mid U_1)$
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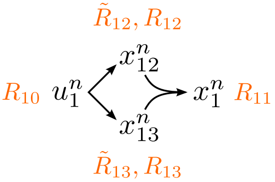

From interference decoding

r_{21}	c_{21}
0	X_{21}
$\min\{\tilde{R}_{21}, H(X_{21} \mid U_2)\}$	U_2
$\min\{R_{20} + \tilde{R}_{21},$ $R_{20} + H(X_{21} \mid U_2), H(X_{21})\}$	$\emptyset$

r_{31}	c_{31}
0	X_{31}
$\min\{\tilde{R}_{31}, H(X_{31} \mid U_3)\}$	U_3
$\min\{R_{30} + \tilde{R}_{31},$ $R_{30} + H(X_{31} \mid U_3), H(X_{31})\}$	$\emptyset$

$R_{20}\quad u_2^n \longrightarrow x_{21}^n\quad \tilde{R}_{21}$

$R_{30}\quad u_3^n \longrightarrow x_{31}^n\quad \tilde{R}_{31}$

3-DIC inner bound — Conditions for $\mathscr{R}_1$ (first receiver)

Condition (example):

$$
\begin{aligned}
\tilde{R}_{13} + R_{11} + \min\{ & R_{20} + \tilde{R}_{21} + \tilde{R}_{31}, \\
& R_{20} + \tilde{R}_{21} + H(X_{31} \mid U_3, Q), \\
& R_{20} + \tilde{R}_{31} + H(X_{21} \mid U_2, Q), \\
& R_{20} + H(X_{21} \mid U_2, Q) + H(X_{31} \mid U_3, Q), \\
& \tilde{R}_{31} + H(X_{21} \mid Q), \\
& H(S_1 \mid U_3, Q)\} \quad \leq \quad H(Y_1 \mid U_1, X_{12}, U_3, Q) \\
& \qquad\qquad\qquad\qquad\qquad + I(X_{12}; X_{13} \mid U_1, Q)
\end{aligned}
$$

3-DIC inner bound — Conditions for $\mathscr{R}_1$ (first receiver)

Condition (example):

$$
\begin{aligned}
\tilde{R}_{13} + R_{11} + \min\{ & R_{20} + \tilde{R}_{21} + \tilde{R}_{31}, \\
& R_{20} + \tilde{R}_{21} + H(X_{31} \mid U_3, Q), \\
& R_{20} + \tilde{R}_{31} + H(X_{21} \mid U_2, Q), \\
& R_{20} + H(X_{21} \mid U_2, Q) + H(X_{31} \mid U_3, Q), \\
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& \qquad\qquad\qquad\qquad\quad + I(X_{12}; X_{13} \mid U_1, Q)
\end{aligned}
$$

- 45 conditions of this type, plus 5 encoder conditions
- Not hard to evaluate by computer

Example conditions for $\mathscr{R}_1$ (first receiver)

$$
\begin{aligned}
\tilde{R}_{13} + R_{11} + \min\{ & R_{20} + \tilde{R}_{21} + \tilde{R}_{31}, \\
& R_{20} + \tilde{R}_{21} + I(S_1; S_1' \mid X_2, U_3, Q), \\
& R_{20} + \tilde{R}_{31} + I(S_1; S_1' \mid U_2, X_3, Q), \\
& R_{20} + I(S_1; S_1' \mid U_2, U_3, Q), \\
& \tilde{R}_{31} + I(S_1; S_1' \mid X_3, Q), \\
& I(S_1; S_1' \mid U_3, Q)\} \ \le \ I(X_1, X_2, X_3; Y_1 \mid U_1, X_{12}, U_3, Q) \\
& \qquad\qquad\qquad\qquad + I(X_{12}; X_{13} \mid U_1, Q)
\end{aligned}
$$

- Saturation interpretation still holds
- Conditional noisy channel capacities replace typical set size

Example conditions for $\mathscr{R}_1$ (first receiver)

$$
\begin{aligned}
\tilde{R}_{13} + R_{11} + \min\Big\{ & R_{20} + \tilde{R}_{21} + \tilde{R}_{31}, \\
& R_{20} + \tilde{R}_{21} + H(X_{31} \mid U_3, Q), \\
& R_{20} + \tilde{R}_{31} + H(X_{21} \mid U_2, Q), \\
& R_{20} + H(X_{21} \mid U_2, Q) + H(X_{31} \mid U_3, Q), \\
& \tilde{R}_{31} + H(X_{21} \mid Q), \\
& H(S_1 \mid U_3, Q) \quad \Big\} \;\le\; H(Y_1 \mid U_1, X_{12}, U_3, Q) \\
& \qquad\qquad\qquad\qquad\qquad + I(X_{12}; X_{13} \mid U_1, Q)
\end{aligned}
$$

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- Conditional noisy channel capacities replace typical set size